**COMPLEX NUMBERS**

The complex number system was constructed to solve problems like .

The general form of a complex number is or . is called the imaginary unit. The above form of the complex number is called the **rectangular form** as will be seen later on.

MAKE UP OF A COMPLEX NUMBER

A complex number is made up of two parts i.e. the real part and he comlex part.

Given a complex number , the real part of z represented as or is the part without the imaginary unit (which is a) and the imaginary part represented as or is the part with the imaginary unit.

The value stands for the **complex variable**.

From the above it can be seen that all real numbers are complex numbers but not all complex numbers are real numbers.

If a complex number, , it is a purely real complex number

If a complex number, , it is a purely imaginary complex number

COMPLEX NUMBER OPERATIONS

1. Addition

2. Subtraction

3. Multiplication

4. Division

CONJUGATE OF A COMPLEX NUMBER

If the complex number z,its conjugate can be represented by .

ADDITIVE INVERSE OF A COMPLEX NUMBER

The additive inverse of a complex number is equal to .

OPERATIONS WITH CONJUFATE

1. . The conjugate of the conjugate of a complex number gives the complex number.

2.

3.

4. If , z is purely real

5.

6.

MODULUS OR ABSOLUTE VALUE OF A COMPLEX NUMBER

The absolute value of the complex variable , written as is given as

Given that are complex numbers

ARITHMETIC OF the complex number

From the above, it can be seen that there is a cyclic effect for the complex numbers.

In general, when k = 0,1,3,4,… we have the following formula for the cyclic effect of multiplying by

, , ,

ARGAND DIAGRAM

The graph of a complex number is called an argand diagram. For simple representation, this diagram is similar to the our normal x and y axes graphs. However, the x-axis is now called the **real axis** and the y-axis is now called the **imaginary axis**. The angle it makes with the positive x-axis (moving in an anti-clockwise direction) is called the **argument**. More light will be thrown on this.

Complex numbers can be expressed in different forms:

1. Cartesian form or rectangular form

2. Polar form

3. Exponential form

All real numbers are complex numbers and a real number “n” can be written as a complex number “n + 0i.

Take for example, comparing real and complex numbers

1. If , then find z

= z + 2(1+i)

let

The value of |z+1| will be a real number. To convert it to a complex number, we can add 0i to it

On comparing,

But

**COMPLEX NUMBERS IN RECTANGULAR FORM**

To graph a complex number in rectangular form, the x-axis is the real axis (that is the values for a) while the y-axis will be the imaginary axis (that is the values for b)

The absolute value of a complex number is given as

The value of this absolute value is always positive

Trying to draw this as a right angled triangle, the hypotenuse will be the absolute value of z.

The angle between |z| and a is

CONJUGATE OF A COMPLEX NUMBER

If the complex number z,its conjugate can be represented by .

ADDITIVE INVERSE OF A COMPLEX NUMBER

**COMPLEX NUMBERS IN POLAR FORM**

When represented in polar form graphically, we have to take note of the r-values. These r values are usually the circles in the graph. The closest circle has an r value of 1, the second closest has an r-value of 2 and so on.

Trying to draw this as a right angled triangle, the hypotenuse will be r.

is said as the argument of z and it is written as or the **amplitude** of the complex number

Remember that when considering the angle, that we have to take note of which quadrant it is found.

For quadrant 1

For quadrant 2

For quadrant 3

For quadrant 4

Therefore the answer for this will be given as

You can still decide to change it to radians

We often use a shorthand version to denote polar form

**MULTIPLYING COMPLEX NUMBERS IN POLAR FORM**

**QUOTIENT OF TWO COMPLEX NUMBERS**

So from the above, it can be noted that the angle used for calculation depends on the reference angle and the position of the reference angle.

**INVERSE OF A COMPLEX NUMBER**

**COMPLEX NUMBERS IN EXPONENTIAL FORM**

In the polar form of complex numbers,

According to the **Euler’s Formula**

Complex Numbers in Exponential Form

In the polar form of complex numbers,

According to the Euler’s Formula

r is the distance between the polar coordinates and the pole (or origin) (0, 0)

is said as the argument of z and it is written as

In polar form, we get an infinite number of possible exponential form of a given complex number.

Each differs by a multiple of

Like…

Therefore, the exponential form can be written as

If , then

From this questions

For every value of theta, you will get the same thing

, , , … will all represent the same complex number

**CONVERTING FROM EXPONENTIAL TO POLAR**

**MULTIPLICATIVE INVERSE**

In exponential form:

then,

This is the multiplicative index in exponential form

Now, by Euler’s formula

Multiplication in exponential form

Let

**DE MOIVRE’S THEOREM**

**Finding the nth power of a complex number**

**FINDING COMPLEX ROOTS OR ROOTS OF COMPLEX NUMBERS**

**ROOTS OF UNITY**

The roots of unity is a number which is complex in nature and gives 1 when raised to the power of a positive integer n. It is also called as “De Moivre system”

The cube roots of unity are

, ,

To prove,

The cubed roots of unity are the cubed roots of 1

Recall,

**PROPERTIES OF CUBE ROOTS OF UNITY**

1. One imaginary cube root of unity is the square of the other

If the cubed roots of unity are

Then,

Therefore, the cubed roots of unity can be written as:

Also,

2. If the two imaginary cubed roots are multiplied, then the product we get is equal to 1.

If the cubed roots of unity are

Then,

3. The sum of the cubed roots of unity is equal to zero.

If the cubed roots of unity are

**Solve the following questions:**

1. Plot each complex number

A. z = 4 + 3i

B. z = -2 – 3i

C. z = 2

D. z = -3

E. z = 4i

F. Z = -3i

2. Calculate the absolute value of each complex number shown below

A. z = 3 + 4i Answer: 5

B. z = 4 – 6i Answer:

C. z = 3 Answer: 3

D. z = -4 Answer: 4

E. z = -8i Answer: 8

3. Write the complex number in polar form

A.

Solution:

To convert it to radian forms,

B.

Solution:

Notice that even though the value of b is negative, we used it as a positive value.

When the graph is drawn, it can be seen that the value of r is the fourth quadrant.

However, in our calculation, we want the angle with the positive x-axis

That will be .

For the last example

,

It can be expressed like

C. . Answer:

D. . This can be easily solved:

If , then

Then if we place this on the graph, we will see that the angle is 0. Therefore if given just one value, the angle will either be 0, 90, 180 or 270

E z = -4

For this, r = 4 and

F. Answer: r = 2,

G. Answer: r = 5,

4. Write the complex number in rectangular form

This can easily be done by plugging in the values

A. Answer: z = 4i

B. Answer:

C. Answer:

5. Write the complex number in rectangular form. Round your answer to the nearest hundredth

A. This calculation should be done in radian mode in your calculator

Answer: To the nearest hundredth.

6. Find the product of the two complex numbers. Write the answer in polar form

A.

Answer:

B.

Answer:

C.

Answer:

D.

Answer: r = 65,

7. Find the quotient of the complex numbers shown below. Write the final answer in polar form

A.

Answer: 4[{cos{50}}+i{sin{50}}]

B.

Answer:

C.

8. Find the quotient z1/z2 of the complex numbers shown below. Write the final answer in polar form using an angle between 0 and 360 degrees

z = 3L210

QUESTIONS

1. Given thatand. Find the values of a and b

2. If z is a complex number such that , then find z

3. If and , evaluate .

4. If , find

5. Find the values of x and y if is the conjugate of

6. If , find

AGENDA

1. The real and complex numbers

2. Representations and Algebra of Complex numbers

3. Complex Functions

4. Roots of Unity

5. De Moivre’s Theorem and Applications